

# The Fibonacci code behind super strings and $P$ -Branes. An answer to M. Kaku's fundamental question

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## Abstract

In “Beyond Einstein” the leading string theoretician and notable science writer Michio Kaku referred to what he labelled the ‘strange’ link between the  $E_8$  exceptional Lie group and the various dimensionalities of strings and super string theories and commented on that by saying “*If we could understand why the numbers 8, 10 and 26 continually crop up in super string theory, perhaps we could understand why the universe is four dimensional*”.

In the present work we demonstrate the existence of a Fibonacci code-like connection between the various coupling constants, charges and dimensionalities of super strings and  $P$ -Brane theories. This code is based on the Fibonacci numbers and the golden mean and in the final analysis, may be attributed to the deterministically chaotic nature of the hyperbolic Cantorian sets fixing the geometry and topology of quantum spacetime.

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## 1. Introduction

One of the most mysterious aspects of super and Heterotic string theories, besides the high spacetime dimensionality required, is the fact that the  $E_8 \otimes E_8$  exceptional Lie symmetry group is essential for its mathematical formulation [1]. This prompted much research to enquire about a possible explanation for a link between  $E_8 \otimes E_8$ , the dimensionalities of strings, super strings and M theory. For instance M. Kaku in Ref. [1] exclaimed about what he calls strange numerics and writes

“...the strangest class of Lie groups however are the  $E(N)$  groups... The weird feature of these groups is that ... the highest value  $N$  can take is 8. .... This  $E(8)$  group is one of the symmetries of super strings. Because 8 is the maximum number that can be constructed, a strange form of “numerology” is emerging, which is intimately linked to the 26 dimensions found in string model and the ten dimensions found in the superstring”.

Kaku goes on to say [1]

“The origin of this “numerology” is unknown even to mathematicians”.

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Finally Kaku expresses his hope in the form of a conditional fundamental question or if, then proposition by saying [1]

“If we could understand why the numbers 8, 10 and 26 continually crop up in super string theory, perhaps we could understand why the universe is four dimensions”.

We could naturally extend the list of strange “numerology” by asking why M theory is eleven dimensional and why the corresponding number of particle-like states is 528 which happens to be exactly  $(2)(16) = 32$  larger than the order of  $E_8 \otimes E_8$ , namely the 496 constituting the number of massless gauge bosons [2].

The preceding words of Kaku are of course symptomatic of what all critical minds write about super string theories, but we have chosen to quote M. Kaku verbatim because he is both a leading string theoretician as well as a gifted science writer [1,2]. Therefore his comments are as deep as they are simple and have evoked two conflicting feelings in the author. To start with Kaku’s comments are absolutely correct and go to the heart of the matter without any superfluous high-notch mathematical arguments. On the other hand, extensive explanation for this so called strange numerology was given in many papers published by many authors, including the present one using a theory which is largely based on a connection between a generalized Fibonacci numbers system and certain chaotic Cantor sets with a Hausdorff dimension equal to the golden mean [3–7]. In turn the golden mean is the limit towards which the Fibonacci ratios of D. Brown’s best seller “The Da Vinci Code” converge [8].

In what follows we seek to show, in the most simplistic and direct way possible, that all these numbers, dimensions and coupling constants of string theory are not numerology and do indeed depend upon and are derivable from the geometry and topology of spacetime [3–7,9–11,4–35].

## 2. The Da Vinci Code and the Fibonacci connection

A central piece of the plot in Dan Brown’s best selling thriller “The Da Vinci Code”, recently made into a film, is revealed in the following abbreviated, recounted dialogue [8]:

“Jacques Saunier’s final communication seemed as unlikely a departing message as any Langdon could image. The message read

13–3–2–21–1–1–8–5  
Oh, Draconian devil!  
Oh, Lame Saint!

Although Langdon had not the slightest idea what it meant, he did understand Fache’s instinct that the pentacle had something to do with devil worship.

... She pulled a scrap of paper from her sweater pocket and handed it to Fache. “Here is the decryption”. Fache looked at the card

1–1–2–3–5–8–13–21

This is it? he snapped. All you did was to put the numbers in increasing order! ... une plaisanterie numerique? ... This is the Fibonacci sequence she declared. ...

Mathematician Leonardo Fibonacci created this succession of numbers in the thirteenth century. Obviously there can be no coincidence that all of the numbers Saunière wrote on the floor belong to Fibonacci’s famous sequence.

Now the Fibonacci series [12–16,34] has of course nothing to do with devil worship and was conceived initially to describe the multiplication of rabbits. Meantime the Fibonacci ratio and its limit, the golden ratio have come up in virtually all conceivable mathematical, physical, chemical and biological disciplines [26,28,34]. In particular the Fibonacci series and the golden mean plays a profound role in the construction of a relatively new spacetime theory, frequently referred to as  $\epsilon^{(\infty)}$  Cantorian-fractal spacetime or *E*-infinity theory [2–7,9–11].

Let us start by showing the intimate connection between the Fibonacci series and nature at its most fundamental level connected not only to electromagnetic fine structure constant but also the dimensions of string theory.

The first thing we do, as shown in the preceding extracts of the Da Vinci Code, is to take 1 and 1 as seed and add every two successive numbers after that. That way one finds the Fibonacci series [12–14,34]

$$F_i = F_{i-2} + F_{i-1}$$

For  $F_{i-2} = F_{i-1} = 1$  one finds

$$1, 1, 2, 3, 5, 8, 11, 21, \dots$$

We note that the ratio of each successive two terms converges towards the golden mean

$$\frac{F_{i-1}}{F_i} \rightarrow \phi = \frac{\sqrt{5} - 1}{2} = 0.618033989 \dots$$

as  $i \rightarrow \infty$ . Now let us take 2 and 2 as seeds. That way one finds [15,16]

$$2, 2, 4, 6, 10, 16, 26, 42, 68, 110, \dots$$

If acquainted with super string theory, then the attentive reader may notice that the ‘in reverse’ order of appearance [16]

$$26, 16, 10, 6 \text{ and } 4$$

are the fundamental dimensions of D. Gross’ Heterotic string theory [2]. In particular we recognise  $D^{(6)} = 6$  as the compactified dimensions and  $D^{(4)} = 4$  as our familiar  $3 + 1 = 4$  dimensional spacetime. In fact one could go two steps further and suspect that 42 is the coupling constant of non-super symmetric GUT unification while  $(2)(68) = 136$  is quite close to the integer value of the inverse electromagnetic fine structure constant  $\bar{\alpha}_0 = 1/137$ .

In what follows we would like to show how the exact electromagnetic fine structure constant may be found using the preceding Fibonacci linear cellular automaton-like methodology [16]. In addition we will show the connection between string theory,  $E_8$  Lie exceptional group and various other physical quantities. Subsequently we will explain the existence of these relations using the very geometry and topology of spacetime as claimed by  $E$ -infinity theory [3–11,14–28,35].

### 3. The exact fine structure constant $\bar{\alpha}_0$ and its relation to the dimensionalities of string theory

Let us use as seed not 2 and 2 as done in the previous section, but with  $2 - 3k$  and  $2 + 2k$  where  $k$  is a small number compared to unity which will be specified later on. Proceeding according to the Fibonacci prescription, one finds

$$\begin{aligned} &2 - 3k \\ &2 + 2k \\ &4 - k \\ &6 + k \\ &10 \\ &16 + k \\ &26 + k \\ &42 + 2k \\ &68 + 3k \\ &110 + 5k \end{aligned}$$

and so on. For reasons which will be explained shortly, let us take  $k = \phi^3(1 - \phi^3) = 0.18033989 \dots$  and write once more the exact value for the ninth Fibonacci number. That way one finds [16]

$$\begin{aligned} 2(68 + 3k) &= 2(68 + 0.54101967) \\ &= 137.0820393 \\ &= \bar{\alpha}_0 \end{aligned}$$

Surprisingly, or as we will see later, not so surprisingly, we obtained the exact theoretical  $E$ -infinity value of the inverse fine structure constant. To find the experimental value we need a certain projection of  $\bar{\alpha}_0$  to find that [3,25]

$$\begin{aligned} \bar{\alpha}_0(\text{experimental}) &= \frac{\bar{\alpha}_0 - k_0}{\cos(\pi/\bar{\alpha}_0)} \\ &= \frac{137}{\cos(\pi/137 + k_0)} \\ &= 137.0359852 \end{aligned}$$

in complete agreement with the precise experimental measurement [25].

By any standard this is a remarkable result but the Fibonacci magic does not stop here. Let us reverse the preceding systematic deduction and start from  $(\bar{\alpha}_0/2)$  of  $E$ -infinity, then scale it down using the golden mean scaling of  $E$ -infinity [3]. Proceeding in this way one finds [3,25]

$$(\bar{\alpha}_0/2)(\phi) = 42 + 2k$$

$$(\bar{\alpha}_0/2)(\phi^2) = 26 + k$$

$$(\bar{\alpha}_0/2)(\phi^3) = 16 + k$$

$$(\bar{\alpha}_0/2)(\phi^4) = 10$$

$$(\bar{\alpha}_0/2)(\phi^5) = 16 + k$$

$$(\bar{\alpha}_0/2)(\phi^6) = 6 + k$$

$$(\bar{\alpha}_0/2)(\phi^7) = 4 - k$$

$$(\bar{\alpha}_0/2)(\phi^8) = 2 + 2k$$

$$(\bar{\alpha}_0/2)(\phi^9) = 2 - 3k$$

This is exactly the same result obtained using the Fibonacci additions, only in reverse order. The value  $26 + k$ ,  $16 + k$ ,  $10$ ,  $6 + k$  and  $4 - k$  are the dimensions of a transfinite version of Gross Heterotic string theory and reduce to it when setting  $k = 0$  compared to unity. Notice that both theories touch each other at  $D^{(10)} = 10$  which is exactly ten dimensional with or without  $k$ .

It is now time to explain the meaning of including  $k$  in the preceding calculation. This  $k = \phi^3(1 - \phi^3)$  is obviously an irrational number and adding  $k$  and similar quantities is termed compactification or transfinite corrections as will be explained in the next section [3].

#### 4. Transfinite correction and their mathematical, geometrical and physical meaning

What happens when one cuts say a square of paper in various triangles and then attempts to reassemble them into another shape, say a rectangle as discussed for instance in [29]. The lesson one gains from such an exercise is that unless the square is golden mean proportioned, we cannot in general transform a square into a rectangle using triangular dissection. Such area preserving transformation, where there is no over lapping and no gaps left, is only possible when transfiniteness of the irrational length is involved [6,29]. It seems somewhat paradoxical that we must fix an irrational length which is practically impossible in order to get the correct answer. The transfiniteness acts here in a way similar to the mortar of a discrete brick wall. Without the mortar there will be unavoidable gaps between the integer bricks. As far as the golden mean is concerned, the following example may clarify what we are trying to say. The exact expression for  $\phi$  is

$$\phi = (\sqrt{5} - 1)/2$$

It is impossible to write  $\phi$  exactly in a rational or a decimal form. Approximately  $\phi$  may be given as

$$\phi = 0.618033989 \dots$$

and we could go on writing numbers after the dot indefinitely. However  $\phi$  could be split into two parts, a rational  $1/2$  and an irrational part  $(1 + k)/10 = 0.118033989 \dots$ . Consequently we have

$$\phi = (1/2) + (1 + k)/10$$

It is this  $k$  which is needed in the scaling of  $(\bar{\alpha}_0/2)$  as well as in the seed for the Fibonacci series leading also to  $\bar{\alpha}_0/2$ . However this does not yet explain the physical relevance of  $k$  nor area preserving transformation. The explanation for that is naturally the conservation of energy which is one of the few main principles upon which our entire physical theories are based. A system for which the phase space shrinks is a dissipative system. By contrast, when the phase space expands, the system is a flutter set and highly unstable. A Hamiltonian system such as we have in quantum mechanics must be symplectic [29]. That means area preserving. The lack of frictional dissipation in this case is compensated for by the irrationality of the winding number. In other words, it is again the most irrational number  $\phi$  which gives the system stability as explained in detail in KAM theorem of nonlinear dynamics [3,6]. Maybe the following geometrical picture could be of help in understanding how transfiniteness appears in the tiling of space.

We start by asking ourselves if it is possible to tile the square by progressively smaller squares. The well known answer is of course, yes and the simplest form of such tiling is the golden mean spiral shape (see Fig. 1) which needs infinitely many squares as shown for instance by Mauldin [3]. A similar situation arises when we add progressively small-

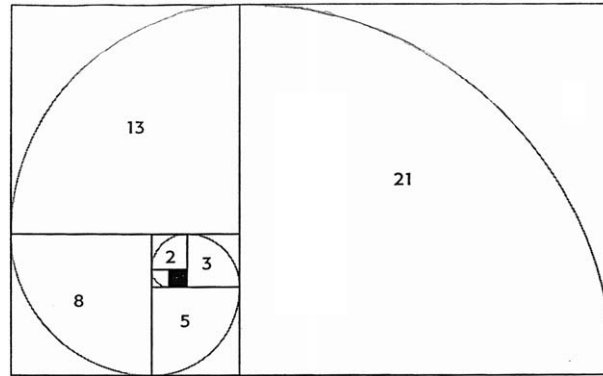


Fig. 1. The golden mean spiral. The squares form Fibonacci numbers. There is an intimate connection between this golden mean spiral and the Mauldin–Williams random triadic set which possesses a Hausdorff dimension equal to the golden mean. These cross connections go on to involve Penrose tiling and *E*-infinity theory. In fact quantum gravity may be envisaged within this picture as a golden mean fractal spacetime Penrose tiling in analogy to Regge calculus. The universe appears very complex. Never the less the rule of the game may be as simple as a Fibonacci-like cellular automata (see Danil Mauldin R. In: Cooper NG, editor. Probability and nonlinear systems in “from cardinales to chaos”. Cambridge: Cambridge University Press; 1989).

ler triangles to Klein’s well known modular curve  $\Gamma(7)$  while preserving the same manifest symmetry. To do that we need infinitely many triangles with a final weight equal to  $16k$  which must be added to the original 336 triangles of  $\Gamma(7)$  giving a final weighted expectation number equal to [3,25]

$$\begin{aligned} \dim \Gamma_c(7) &= \Gamma(7) + 16k \\ &= 336 + (16)(0.18033989) \\ &\simeq 339 \end{aligned}$$

We may mention on passing that the distribution involved in the preceding calculation is a discrete gamma distribution [3]. This is a distribution from the same family of Planck distributions only completely discrete and is the same distribution upon which *E*-infinity spacetime theory is based [3,6,25]. There is an important consequence for introducing quantities such as  $k$  to our theory. These quantities which correspond to a fractal form of geometry make our theory fuzzy in a mathematically precise way. This is not a contradictory statement. It is exact and fuzzy in the same sense as theory of probability delivers exact expectation values.

It is interesting to note that Riemann curvature tensor in  $n = 8$  dimensions have the same number of independent components as  $\dim \Gamma(7)$  namely [25]

$$\begin{aligned} R^{(n=8)} &= n^2(n^2 - 1)/12 \\ &= 336 \end{aligned}$$

Similarly in the ten dimensions of super string theory, the sphere kissing number is [21]

$$K_s^{(n=10)} = 336$$

In addition the order of the linear simple group  $SL(2, n)$  is given by the same number for  $n = 7$

$$\begin{aligned} |SL(2, 7)| &= n(n^2 - 1) \\ &= (7)(48) \\ &= 336 \end{aligned}$$

Consequently we could postulate a transfinite compactified version for  $R^{(n)}$ ,  $K_s^{(n)}$  and  $SL(2, n)$  for which  $336 \rightarrow 336 + 16k \cong 339$ .

It is extremely simple to show how all of these values are scaling of  $(\bar{\alpha}_0/2)$  and deeply interwoven with string theory and we just need to divide  $336 + 16k$  by 8 and then multiply with  $1/\phi$  and find

$$\begin{aligned} \left(\frac{336 + 16k}{8}\right) \left(\frac{1}{\phi}\right) &= (42 + 2k)(1/\phi) \\ &= 68.54101939 \\ &= \bar{\alpha}_0/2. \end{aligned}$$

In the next section we will show that the above quoted value  $42 + 2k$  is nothing but 10 copies of  $\sim \langle n \rangle = 4 + \phi^3 = 4.236067977$  which is the Hausdorff dimension of quantum  $E$ -infinity spacetime [3]. In Fig. 1, we give some intuitive arguments for the exact mathematical relation between the golden mean, Penrose tiling, Regge calculus and the Hausdorff dimension of the random Cantor sets of  $E$ -infinity theory [3–7,9–11].

### 5. The link between $E$ -infinity space, the four dimensionality of the classical spacetime, super strings and Lie exceptional symmetry group $E_8$

We have seen in the previous sections how the Fibonacci series and the golden mean links the electromagnetic fine structure constant with the Heterotic super string theory. Never the less, one can still demand an explanation of why this is the case, all apart of the unreasonable effectiveness of mathematics [30]. In the view of  $E$ -infinity theory, the final explanation may be found in the very structure of spacetime. More accurately, in the geometry and topology of macro and micro-spacetime. This point was discussed in many previous papers but at the risk of being a little repetitive, we recall that while on our scale in the classical world space and time seem to be decoupled, smooth and linear at very large velocities comparable to that of light, space and time become spacetime as in the special theory of relativity. In addition, at very large cosmic distances, spacetime curves as is the case with the general theory of relativity. Finally at very small distances relevant to quantum mechanics, spacetime is neither smooth nor continuous. In fact as we come nearer to the Planck length ( $\ell_p \simeq 10^{-33}$  cm) spacetime may be likened to an infinite dimensional stormy ocean [3,20]. In  $E$ -infinity we set out following some pioneering ideas due to J.A. Wheeler, K. Menger and D. Finkelstein to construct spacetime ab initio.

The final result was a resolution dependent spacetime manifold. Seen at classical resolution, this manifold is indistinguishable from our  $3 + 1$  spacetime. For cosmic distance however, the manifold is curved. However the crucial point is that at the quantum resolution, this spacetime is a random infinite dimensional but hierarchal Cantor set. The test which this manifold must pass is to be able to give the impression of a smooth four dimensional spacetime at the classical resolution corresponding to Newtonian mechanics. To show that this is indeed the case, we sum all of the  $n$  possible dimensions from zero to infinity, give each  $n$  a weight equal to  $(d_c^{(0)})^n = \phi^n$  where  $d_c^{(0)} = \phi$  is the Hausdorff dimension of a random one dimensional Mauldin–Williams triadic set [3]. This way one finds that [3,20]

$$\begin{aligned} \sim \langle n \rangle &= \sum_{n=0}^{n=\infty} n \phi^n \\ &= 4 + \phi^3 \\ &= 4.236067977 \end{aligned}$$

Thus although our space is formally infinite dimensional ( $n_f = \infty$ ), it still has a finite expectation value for these dimensions, namely  $\sim \langle n \rangle = 4 + \phi^3$ . In Figs. 1–3, we have attempted to guide the reader to the way in which he or she could visualize this space. In addition from one important formula of  $E$ -infinity called the bijection formula, we know that [3,20,29]

$$d_c^{(n_t)} = \left( \frac{1}{d_c^{(0)}} \right)^{n_t-1}$$

where  $n_t$  is the topological dimension and  $d_c^{(0)} = \phi$ . Setting  $n_t = 4$  one finds

$$\begin{aligned} d_c^{(4)} &= \left( \frac{1}{\phi} \right)^{4-1} \\ &= 4 + \phi^3 \end{aligned}$$

In other words, the topological dimension associated with the Hausdorff dimension  $\sim \langle n \rangle = 4 + \phi^3$  is  $n_t = 4$ . Thus at our resolution the  $E$ -infinity spacetime gives the impression of a  $n_t = 4$  manifold. That means we are living in the topological expectation value  $n = 4$  while quantum mechanics inhabits spacetime regions with higher dimensionality depending on the energy scale which we are considering (see Figs. 1–4). Now we can express  $\bar{\alpha}_0$  in terms of the expectation value of the Hausdorff dimension of our spacetime, namely

$$\begin{aligned} \bar{\alpha}_0 &= (20)(\sim \langle n \rangle)(1/\phi) \\ &= 137 + k_0 \\ &= 137.082039325 \end{aligned}$$

where  $k_0 = \phi^5(1 - \phi^5)$ .

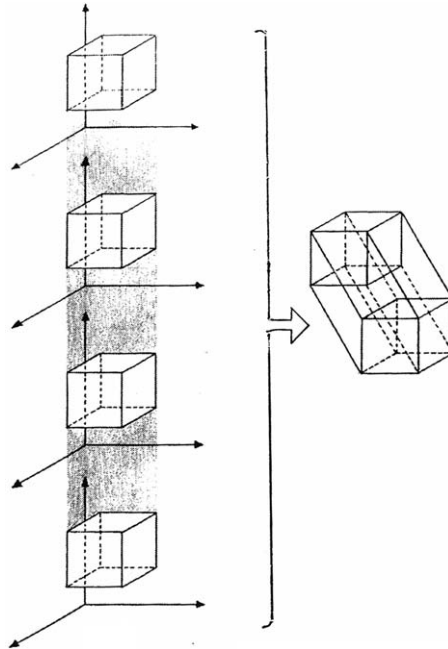


Fig. 2. When a zero dimensional space called point moves it creates a one dimensional path behind it called line. When the line moves it sweeps a world sheet called surface. By analogy the surface creates a volume, say a cube and then the path of a cube is a four dimensional hyper cube as shown in this figure. We need not stop here. The path of a four dimensional cube can be interpreted as a five dimensional cube similar to that shown in Fig. 4 and so on ad infinitum. *E*-infinity spacetime is created in a similar way, however each higher dimensional object is smaller than the previous one. Thus it is hierarchal infinite dimensional space. Never the less, seen from afar it looks as if it were four dimensional.

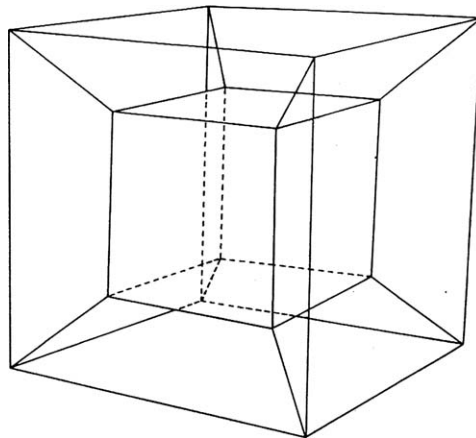


Fig. 3. The projection (i.e. shadow) of a four dimensional hyper cube (i.e. a tesseract) on our classical three dimensional space would look like a cube enveloping a smaller cube. Now we could go on imagining higher dimensional cubes and the resulting projections should then, by analogy, appear as a cube enveloping another cube which is again enveloping another cube and so on indefinitely. This is quite similar to a Russian doll or in fact our continued fraction representation of the expectation value of the Hausdorff dimension of *E*-infinity Cantorian spacetime.

$$\sim \langle n \rangle = 4 + \frac{1}{4 + \frac{1}{4 + \dots}}$$

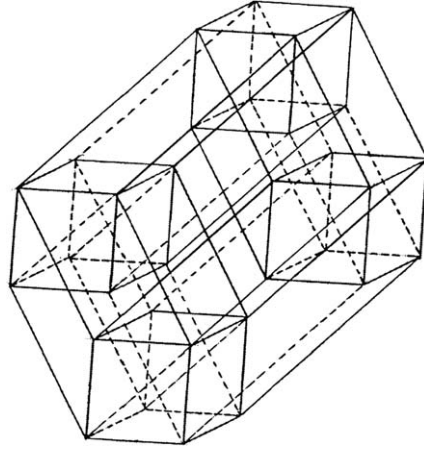


Fig. 4. A five dimensional cube.

Next let us show that the order of  $E(8)$ , that means the dimension, is intimately linked to  $\bar{\alpha}_0$  by similar golden mean scaling. Thus we have

$$\begin{aligned} \dim(E(8)) - (k^2) &= (\bar{\alpha}_0/2)(3 + \phi) \\ &= (68.54101965)(3.618033989) \\ &= 247.987387 \\ &\simeq 248 \end{aligned}$$

exactly as in the classical theory where  $\dim E_8 \otimes E_8 = (2)(248) = 496$  [29].

In the next section we will look at further links between coupling constants, charges and super strings.

## 6. Further links between $P$ -Branes, super strings, coupling constants and the topology of $E$ -infinity spacetime

In all our previous discussion we considered only pure numbers referring to dimensionless couplings and space-time dimensions. However the amazing influence of spacetime stretches out and decides on the mass of elementary particles. A particularly intriguing example of that is the relation between the mass of the electron which is a non-composite elementary particle and the mass of the charged and the neutral meson which is the simplest composite particle consisting only of two quarks. Thus by a simple scaling we can find the mass of the  $\pi^\pm$  meson in terms of  $m_e$  to be [3,29]

$$m_{\pi^\pm} = (2\bar{\alpha}_0 - 1)m_e$$

with  $m_e = 0.511$  MeV one finds

$$\begin{aligned} m_{\pi^\pm} &= (2\bar{\alpha}_0 - 1)0.511 \\ &= (273.1640786)(0.511) \\ &= 139.5868442 \text{ MeV} \end{aligned}$$

in excellent agreement with the experimental value [29]. Similarly one finds for  $\pi^0$  that

$$\begin{aligned} m_{\pi^0} &= (2\bar{\alpha}_0 - 10)(0.511) \\ &= (264.1646678)(0.511) \\ &= 134.9878386 \text{ MeV} \end{aligned}$$

again in full agreement with the experimentally found mass. The amazing point in the preceding calculation however is the two scaling exponents

$$\begin{aligned} \lambda_1 &= (2\bar{\alpha}_0 - 1) \\ &= 273.1640786 \end{aligned}$$

and

$$\begin{aligned} \lambda_2 &= (2\bar{\alpha}_0 - 10) \\ &= 264.1640678 \end{aligned}$$

In the first case  $\lambda_1$  gauged in Kelvin is numerically equal to the absolute temperature  $T_0 = 273.16$  K which was fixed using the triple point of water by international convention and thought to have very little to do with high energy particle physics. In addition  $\lambda_1 = 273$  is equal to the right hand side of the anomaly cancellation condition of string theory. On the other hand,  $2\lambda_2$  gives the number of states in an eleven dimensional 5-Brane theory. This is easily demonstrates by noting that [4,31]

$$\begin{aligned} N_0 &= \binom{11}{1} + \binom{11}{2} + \binom{11}{5} \\ &= 528 \end{aligned}$$

while

$$\begin{aligned} 2\lambda_2 &= 528 + 4k_0 \\ &= 528.3281356 \\ &\simeq 528 \end{aligned}$$

In fact if we add the 20 of  $R^{(4)} = 20$  of classical general relativity and divide by 8, one finds  $\bar{\alpha}_0/2$  but this time as the number of elementary particles  $N(\text{SM})$  to be expected in a minimally extended standard model [29]

$$\begin{aligned} N(\text{SM}) &= (2\lambda_2 + R^{(4)})/8 \\ &= (548)/8 \\ &\simeq 69. \end{aligned}$$

We see the multitude of cross connections is amazing. However they are all inter-connected and related to a single spacetime manifold proportioned according to the Fibonacci number and golden mean scaling. For instance the particle content  $N(\text{SM}) \simeq 69$  could be derived in a formal way using either of two manifolds. The first is termed  $K(\epsilon^{(\infty)})$  which is a fuzzy version of a Kähler manifold known as  $K3$ . The second manifold  $K(\Gamma(7))$  is a fuzzy holographic boundary manifold [32,33]. The union of both manifolds define the electromagnetic fine structure constant. More accurately we have [32,33]

$$\begin{aligned} \bar{\alpha}_0 &= (\tau_1)(\tau_2) \\ &= (13.7082039325)(10) \\ &= 137.082039325 \end{aligned}$$

where  $\tau_1$  is the signature of  $K(\epsilon^{(\infty)})$  and  $\tau_2$  is the signature of  $K(\Gamma(7))$ . Also the elementary particles content is given as the multiplication of the complimentary instanton number with the dimension of the symmetry group divided by the spin. Thus for the classical  $K3$  and using  $\Gamma(7)$  one finds [32,33]

$$\begin{aligned} N(\text{SM}) &= \frac{(\chi)(\dim\Gamma(7))}{\text{spin } 7} \\ &= \frac{(24)(336)}{(2)^7} \\ &= \frac{8064}{128} \\ &= \frac{N_0}{128} \\ &= 63 \end{aligned}$$

where  $\chi = 24$  is the instanton number [25].

Note that  $N_0 = 8064$  is in full agreement with the result quoted in Kaku's book on string theory [2].

Proceeding to the exact transfinite analysis one finds for the bulk as well as for the boundary that [32,33]

$$\begin{aligned}
 N(\text{SM}) &= \frac{(\chi)(\dim E8 \otimes E8)}{\text{spin } 7 + 8k} \\
 &= \frac{(18 - 2\phi^6)(496 - k^2)}{128 + 8K} \\
 &= \bar{\alpha}_0/2
 \end{aligned}$$

where  $\chi = 18 - \phi^6$  is the instanton number of  $\Gamma(7)$  as well as

$$\begin{aligned}
 N(\text{SM}) &= \frac{(\chi)(\dim \Gamma c(7))}{128 + 8k} \\
 &= \frac{(26 + k)(336 + 16k)}{128 + 8k} \\
 &= \bar{\alpha}_0/2
 \end{aligned}$$

respectively, as obtained in previous publications and where  $\chi = 26 + k$  is the instanton number of  $\varepsilon^{(\infty)}$  [32,33].

**7. Ji-huan He’s eleven dimensional Cantorian spacetime theory**

The present work would be incomplete if we did not mention He’s brilliant idea of embedding an eleven dimensional spacetime theory into  $E$ -infinity space [4]. Further development of He’s work leads to [31]

$$\begin{aligned}
 D^{(11)} &= (1/\phi)^5 \\
 &= 11 + k/2 \\
 &= 11 + \frac{1}{11 + \frac{1}{11 + \dots}}
 \end{aligned}$$

where  $k = \phi^3(1 - \phi^3)$  is again our transfinite correction. Using  $D^{(11)}$  it is possible to determine the exact  $N(\text{SM})$  in one step only

$$\begin{aligned}
 N(\text{SM}) &= (6 + k)(11 + k/2) \\
 &= \bar{\alpha}_0/2 \\
 &\cong 69
 \end{aligned}$$

**8. Conclusion**

The strange “numerology” of super strings and related theories are not at all “numerology”. They are rationally explained facts in which mathematics and physics melt. The connection is spacetime geometry and topology and the tools are the Fibonacci series and the golden mean scaling.

Because general relativity has been around for such a long time, no one finds it strange that the curvature of spacetime creates gravity and that mass curves spacetime. In principle there is no reason why electromagnetism and all the other fundamental interactions should not be linked to spacetime in a similar way. If we really believe in unification, then all fundamental forces are the same and must come from the same source, namely spacetime. Super strings was maybe the first theory to take this important step following Einstein’s insight. Loop quantum mechanics and twistors each in a different way, contribute to the understanding of quantum gravity gained from string theory.  $E$ -infinity seems to tie many loose ends created by omitting the transfinite fractal nature of spacetime at the quantum energy scale.

There are two extreme views which one can take with respect to string theory. The first is mostly adopted not by the Masters, but mainly by their students that string theory is perfect, nearly complete and any way, it is the only game in town. The other view which is increasingly heard now a days, may have come as a consequence of frustration with the slow development in connecting the theory with reality, holds that string theory is a mathematical theory with no physical content and that those working on it are shooting in the dark and do not really understand the physical meaning of what they are talking about. I think that both views are over the top. String theory is a beautiful discovery with a great deal of physical meaning. When combining super strings with loop quantum mechanics, twistors and  $E$ -infinity theory, string theory becomes the central piece of contact with reality.

We hope the present work, which was prompted by Kaku’s long standing remarks, has helped to shed light on the depth and physical beauty of both string theory and  $E$ -infinity theory.

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