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String theory, exceptional Lie groups hierarchy and the structural constant of the universe

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Abstract

We introduce a new Magnum Fuzzy Kähler manifold with 548.328 fractal-like states. The work is based on some recent results revealing a curious finite exceptional Lie symmetry groups hierarchy. Those results support strongly claims that with a probability equal to 1, nine elementary particles are still missing from the standard model. The present paper provides further mathematical arguments for the correctness of the claim and links it to the 2D Map of E_8.

1. Motivation and general remarks

Witten motivated his 11 dimensional M Theory in a deceptively simple and remarkably intuitive way. First, he argued correctly that seven space dimensions are needed for embedding the indispensable $SU(3)SU(2)U(1)$ of the standard model. Subsequently, he added our $3+1$ space time dimensions to them and concluded that

$$D = (\mathbf{3} \otimes \mathbf{2}) \oplus \mathbf{1} \oplus \mathbf{4} = 7 + 4 = 11.$$

Following in the footsteps of Witten, a variation on this theme could be the following:

We know that $|E_8| = \mathbf{248}$ is the symmetry group of string theory. We also know that 26 spacetime dimensions are needed for formulating this theory. Consequently, the particle-like dimension could be added together to give:

$$|E_\mathbf{8}| + D^{(\mathbf{26})} = 248 + 26 = 274 = (2)(137) = (2)\bar{\alpha}_0$$

However, we know that superstring theory starts with $E_\mathbf{8}E_\mathbf{8}$ rather than $E_\mathbf{8}$ only. Consequently, the corresponding magnum number of states must be

$$|E_\mathbf{8}| + D^{(\mathbf{26})} + E_\mathbf{8} + D^{(\mathbf{26})} = 548 = (4)(137) = 4\bar{\alpha}_0,$$

where $\bar{\alpha}_0 \simeq 137$ is the inverse of the electromagnetic fine structure constant. Noting that the exact theoretical value of $\bar{\alpha}_0$ is given by

$$\bar{\alpha}_0 = (20)(1/\phi)^\mathbf{4} = 137 + k_0 = 137.082039325,$$

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where $\phi = (\sqrt{\mathbf{5}} - \mathbf{1})/\mathbf{2}$, the preceding result is tantamount to the claim that there should exist a fractal-like Fuzzy Magnum Kähler manifold considerably larger than that corresponding to that of $|E_8 E_8| = 496$ namely

$$|E_\infty E_\infty| = 4\bar{\alpha}_0 = 548 + 4k_0 = 548.3281573 \simeq 548.$$

In various recent papers, a related and highly interesting and partially unexpected result was revealed regarding the telescopic structure of the well-known Lie exceptional groups [1–4]. In fact and as mentioned earlier on, Witten has shown some time ago that embedding $SU(3)SU(2)U(1)$ of the standard model needs seven dimensions which when added to our $3+1$ spacetime results naturally in the 11 dimensions of his M theory [7]. This is just one dimension more than the 10 of superstring theory [6,7]. Consequently, if we start by $D_{\mathrm{m}} = 11$ and proceed upwards until we reach E_8, then we may expect the following hierarchy to add to a maximal total number of states given by [1–4]:

$$D_{\mathrm{m}} + |SU(\mathbf{3})SU(\mathbf{2})U(\mathbf{1})| + |G_\mathbf{2}| + |F_\mathbf{4}| + |E_\mathbf{6}| + |E_\mathbf{7}| + |E_\mathbf{8}| = 11 + 12 + 14 + 52 + 78 + 133 + 248 = 300 + 248 = 548.$$

This is by now a well-known dimension of what we have called earlier on E_∞ symmetry group. In what follows we will be looking at various schemes for extracting physical information from the above with regards to what Ji-Huan He has called the missing 9 particles of the standard model [5,8,9]. We recall that the present author has calculated using a Fuzzy Kähler spacetime manifold the maximum number of elementary particles contained in the standard model and came to the conclusion that $N(SM)\max = \bar{\alpha}_0/\mathbf{2} = \mathbf{69}$ particles [5,8,9].

2. Factoring the standard model out of $|E_8 E_8|$

The role of $|E_8 E_8| = 496$ is well known and understood in heterotic superstring theory. Leaving $|E_8 E_8|$ for a moment and considering a maximally symmetric four manifold in a super space, we see that the dimensionality m must be $m = (4)(8) = 32$. Consequently, the number of states is equal to the number of Killing Vector Field [6–14]:

$$N_k^{(\mathbf{32})} = m(m+1)/2 = (32)(33)/2 = 528.$$

Considering the 32 dimensional space to be quasi-Mikowskian, then one could break the 528 vector fields as follows [10–14]:

$m = 32$ are translations,

$m - 1 = 31$ are boosts

and

$(m-1)(m-2)/2 = 465$ are space rotations.

Notice first that

$$(m-1) + (m-1)(m-2)/2 = 31 + 465 = 496 = |E_\mathbf{8} E_\mathbf{8}|$$

Second we see that

$$N_k^{(\mathbf{32})} = |E_\mathbf{8} E_\mathbf{8}| + 32.$$

In other words $N_k^{(\mathbf{32})} = 528$ could be written in segregated form as:

$$N_k^{(\mathbf{32})} = \text{spin rotation} + \text{boosts} + \text{translation} = 465 + 31 + 32 = 465 + 63.$$

We focus our attention on the boosts and translation part of the above which amounts to

$$b + t = 31 + 32 = 63.$$

One could interpret this number as that found for the elementary particle content of the standard model based on the well-known results of Green, Schwartz, Witten and Gross' heterotic string theory [1]. We recall that the number of massless states was found then to be 8064 when considering that every possible spin direction in $D = 8$ is a different particle. However, when we dispose of this for the standard model unrealistic counting, then we can divide 8064 by the spin representation 128 and obtain [10–14]:

$$N(SM) = 8064/128 = 63.$$

This squares with $b + t = 63$ and hence our interpretation. In the uncompactified version of our E-Infinity theory using the holographic principles and Kähler manifolds, the same results may be obtained as follows:

$$N(\text{total}) = [SL(2,7)](n_{\text{instanton}}) = (336)(24) = 8064$$

Consequently,

$$N(SM) = (8064)/(\mathbf{2})^{\mathbf{8-1}} = (8064)/128 = 63$$

as anticipated.

Now the E-Infinity based Ji-Huan He's assertion states that $N(SM) = 69$ and not 63 [5–9]. Since only 60 particles have been discovered so far, then one is inclined to think that at least some of the 465 "space rotational" particles may materialize as real particles increasing 63 to 69. To argue in favor of that, we will have to resort to an extended exceptional Lie group hierarchy introduced in earlier publications where a convenient notation based on the exceptional Lie groups characteristics of $SO(10)$ and $SU(5)$ Grand Unification Groups leads to $SO(10) = E_5$ and $SU(5) = E_5$. Proceeding this way, one finds:

$$|E_8| + |E_7| + |E_6| + |E_5| + |E_4| = (248 + 133 + 78) + (45 + 24) = 459 + 69 = 528.$$

In particular we see that

$$|E_5| + |E_4| = 45 + 24 = 69 = N(SM)$$

which is the correct integer approximation to $N(SM) \simeq \mathbf{69}$.

This is not yet the most interesting result of this section because we see that the 69 does not all come from the translation and boost interpretation discussed earlier on. In fact we could write our 69 differently as:

$$|E_5| + |E_4| = (|E_5| + SM) + \overline{SM} = (45 + 12) + 12 = 57 + 12,$$

where $\overline{SM}$ are the super symmetric partners of $|SU(\mathbf{3})SU(\mathbf{2})U(\mathbf{1})| = \mathbf{12}$ and 57 happen to be exactly equal to the space which E_8 forms. A two-dimensional projection of this E_8 object is shown in Fig. 1 and one should notice the possible relation to $\Gamma(7)$ where

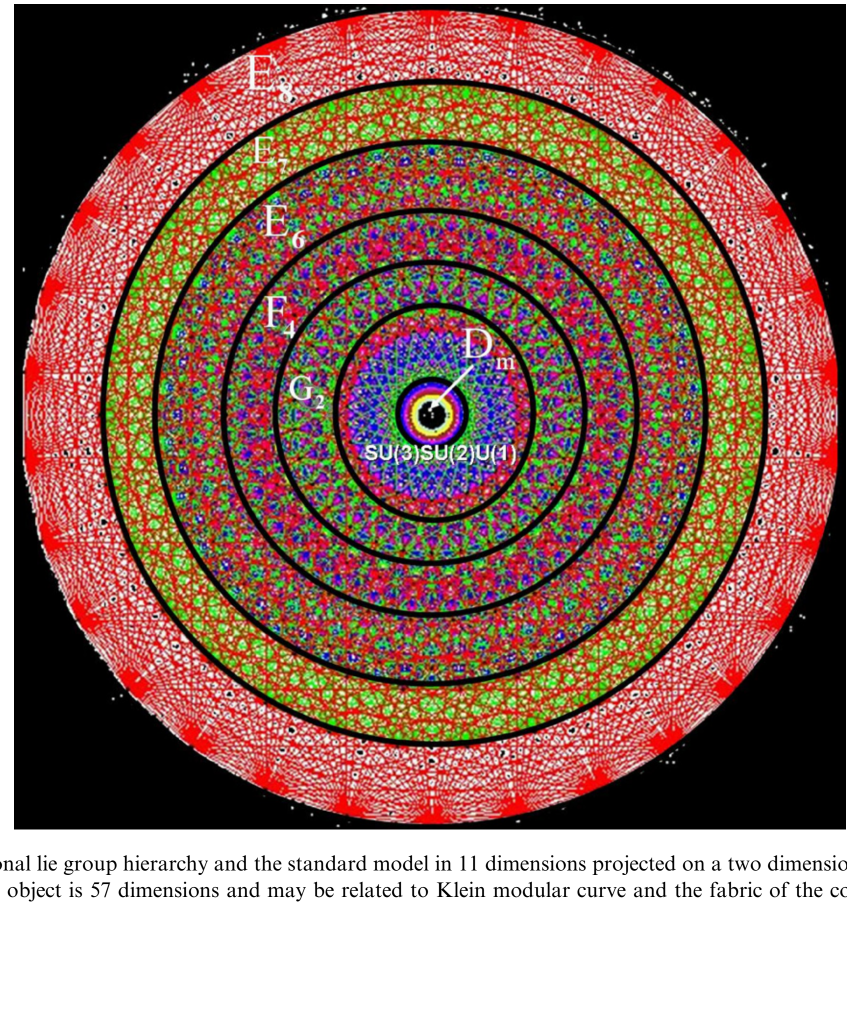

Fig. 1. An exceptional lie group hierarchy and the standard model in 11 dimensions projected on a two dimensional representation of E8. Notes that this object is 57 dimensions and may be related to Klein modular curve and the fabric of the cosmos.

$$\text{Dim}\Gamma(7) = |SL(\mathbf{2},\mathbf{7})| = (6)(56) = 336$$

here 336 are the degrees of freedom of Klein's original modular curve used in deriving $N(SM) = 63$ earlier on.

It could be quite speculative to suggest that the proximity of 56 and 57 is a hint that $\Gamma(7)$ as well as the space shown in Fig. 1 may be strongly related to the structure of our real cosmos but the implication is too tantalizing to be passed on in silence.

3. Segregation via Witten's Brane model

A particularly ingenious and in the present context illuminating model is the $P = 5$ in $D = 11$ Brane theory of Witten [7]. As is well known, the number of states in this case is given by

$$N(B) = \binom{11}{\mathbf{1}} + \binom{11}{\mathbf{2}} + \binom{11}{\mathbf{5}} = 11 + 55 + 462 = 66 + 462 = 528.$$

Clearly the 66 are possible real particles and are related to string and membrane in this interpretation. Since the total number is still 528, then at least 3 more particles in the standard model come from $p = 5$ Branes [7].

At the beginning the value 69 may seem incorrect particularly because a three-step symmetry breaking of 528 leads to

$$N(SM) = 528/8 = 66.$$

However the point is that the largest possible number is not 528 but rather that derived in the introduction namely 548. Consequently we have

$$N(SM) = 548/8 = 68.5 \simeq \mathbf{69}.$$

The preceding analysis suggests furthermore that the following modified hierarchy is a valid model for $N = 528$ namely

$$(|E_\mathbf{8}| + |E_7| + |E_\mathbf{6}|) + \binom{11}{\mathbf{2}} + |G_\mathbf{2}| = 459 + 55 + 14 = 528.$$

Note now the remarkable result

$$\binom{11}{\mathbf{2}} + |G_\mathbf{2}| = 55 + 14 = 69.$$

This may also suggest that $|G_2|$ is related to the low energy regime. In fact, it is well known that the only non-trivial extension of Einstein's equations in 70 years is super-gravity and the symmetry group for this theory is given by [7]:

$$|\text{OSP}(\mathbf{1}/\mathbf{4})| = \mathbf{14} = |G_\mathbf{2}|.$$

In addition, the transfinite extension of $|G_\mathbf{2}|$ could well be

$$|G_\mathbf{2}| \to \mathbf{14} - \mathbf{2}\phi^\mathbf{4} = \bar{\alpha}_0/\mathbf{10} = 13.7082039325,$$

where $\bar{\alpha}_0 \simeq \mathbf{137}$ is the well known low energy scale E-M coupling and $\phi = (\sqrt{\mathbf{5}} - \mathbf{1})/\mathbf{2}$ [10–14].

4. The magnum fuzzy Kähler manifold

We have discussed at considerable length two Fuzzy Kähler manifolds which are an extension of the crisp k_3. We recall that the instanton density of k_3 is $n = 24$ and that the corresponding holographic degrees of freedom are given by 336 of $\Gamma(7)$. That way the familiar result 63 was obtained for $N(SM)$. The transfinite extension of this calculation is given by [11]:

$$N = 24 \to 26 + k = 26.18033989$$

and

$$D = 336 \to 336 + 16k = 336 + 16(0.18033989) = 338.8854382.$$

Consequently one finds,

$$N(SM) = \frac{(\mathbf{26}+k)(\mathbf{336}+\mathbf{16}k)}{(\mathbf{128}+\mathbf{8}k)} = 137.0820392/2 \cong \mathbf{69}.$$

However in using the bulk with $D = 496$, one has to make the following replacement:

$$N = 24 \to 18 - 2\phi^{\mathbf{6}}$$

and

$$D = 496 \to 496 - k^{\mathbf{2}},$$

where $\phi = (\sqrt{\mathbf{5}} - \mathbf{1})/\mathbf{2}$ and $k = 0.18033989$.

Consequently one finds the same result once more

$$N(SM) = \frac{(\mathbf{496}-k^{\mathbf{2}})(\mathbf{18}-\mathbf{2}\phi^{\mathbf{6}})}{\mathbf{128}+\mathbf{8}k} = \bar{\alpha}_0/\mathbf{2} \simeq \mathbf{69}.$$

Now we introduce a new manifold $D = 548 + 4k_0$ and note that the corresponding instanton density of the fuzzy boundary Kähler is $n = 16 + k$. Thus in analogy to the preceding calculation, one finds that

$$N(SM) = \frac{(\mathbf{548}+\mathbf{4}k_0)(\mathbf{16}+k)}{\mathbf{128}+\mathbf{8}k} = \bar{\alpha}_0/\mathbf{2} \simeq \mathbf{69}$$

exactly as before.

The complete cohomology of the two new fuzzy Kählers will be given elsewhere.

5. Discussion

Lie groups distinguish themselves from other symmetry groups by the additional aspect of having a geometrical and topological structure of a manifold. The vital role of Lie groups originates from the work of Klein who demonstrated that geometry is simply the study of symmetry groups. The link to particle physics on the other hand stems from the work of Noether.

The size of a group is measured by its dimension as a manifold which means the number of degrees of freedom involved in fixing an element in the group. If we are confined to compact Lie groups, then we can be sure that there are only four families. In addition there is exactly 5 more exceptional Lie groups namely G_2, F_4, E_6, E_7 and E_8 with dimensions 14, 52, 78, 133, and 248 respectively.

By removing the condition of smooth manifolds, E-Infinity theory introduced transfinite versions of Lie groups and the various dimensions were revised by including transfinite corrections. In so doing, we moved from studying crisp structure to studying fuzzy topology and geometry which is the natural setting for quantum physics.

A particularly revealing example is the following $SL(2,n)$ Lie group. For $n = 8$ one finds

$$|SL(\mathbf{2},\mathbf{8})| = \mathbf{8}(\mathbf{8}^2 - \mathbf{1}) = (8)(63) = 504.$$

This corresponds to the well-known heterotic massless states namely,

$$N = (504)(16) = 8064.$$

By contrast, introducing the transfinite continuation, one finds

$$|SL(\mathbf{2},\mathbf{8})_c| = \mathbf{8}[(\mathbf{8}+\mathbf{2}\phi^{\mathbf{3}})^{\mathbf{2}} - (\mathbf{3}+\phi^{\mathbf{3}})] = (\bar{\alpha}_0\mathbf{12})(\mathbf{8}) = 548.3283728,$$

which is our $|E_\infty|$. Let us dwell a little on the meaning of the preceding extension.

In the classical case 8 is nothing but the $E^{(\infty)}$ version of super pace. Consequently for the extension we used the Hausdorff dimensions, namely $(\mathbf{4}+\phi^{\mathbf{3}}) + (\mathbf{4}+\phi^{\mathbf{3}}) = \mathbf{8}+\mathbf{2}\phi^{\mathbf{3}}$. Second, in the classical case we take off $\mathbf{8}^{\mathbf{2}}$ the dimension of a line boundary. Therefore and by analogy, in the transfinitely extended case, we take off the dimension of the boundary of $E^{(\infty)}$ namely $(\mathbf{4}+\phi^{\mathbf{3}}) - \mathbf{1} = \mathbf{3}+\phi^{\mathbf{3}}$.

Another less involved and more straightforward example is the following:

Consider the simplectic group $sp(n)$ for $n = 8$. The dimension in this case is

$$|sp(\mathbf{8})| = \mathbf{2}n^{\mathbf{2}} + n = (2)(64) + 8 = 136.$$

The transfinite version on the other hand is

$$|sp(\mathbf{8})_0|_c = (\mathbf{2})(\mathbf{8})(\mathbf{8}+k/\mathbf{2}) + (\mathbf{8}-\mathbf{2}k) = 137.082039325 = \bar{\alpha}_0$$

where $\bar{\alpha}_0$ is the electromagnetic fine structure constant.

The importance of the exceptional Lie symmetric group E_8E_8 is well known in high energy physics. There are various remarkable and partially mysterious properties of E_8E_8. For instance dividing $|E_8E_8|$ by the Rank $R = 8$ an estimate of the number of elementary particles of the standard model is easily found

$$N(SM) = |E_8E_8|/R = 496/8 = 62.$$

Since 60 particles have been found already in various refined experiments, the additional 2 particles left could be the Higgs and the graviton. We could determine $|E_i|/R$ for 5 well known exceptional groups as well as the $SO(\mathbf{10}) = E_5$ and $SU(5) = E_4$ unification groups and find that

$$\sum_{i=4}^{i=8} E_i/R_i = 62 + 38 + 26 + 18 + 12 = 156.$$

In other words we have the important relation:

$$\sum_{i=4}^{i=8} E_i/R_i = \| E_6E_6|.$$

This makes E_6E_6 an informal center of gravity for the E_i exceptional family.

6. Conclusion

For the last 10 years at least, the present author has wondered if the compactified version of Klein modular curve could be a representative of the fabric of the universe or at least its real holographic boundary. In a sense Fig. 1 of the two-dimensional representation of the 57 dimensional E_8 which has the rank $R = 8$ and the order 248 could be also the two-dimensional image of the fabric of the cosmos. The present analysis connecting the exceptional Lie symmetry group's hierarchy suggests that this maybe indeed true.

In fact based on the present rather sketchy analysis, it would seem that the electromagnetic fine structure constant is the most fundamental constant in nature and that at the ultra high unification energy, all isometries added together are nothing more than four copies of the inverse of this constant. In other words, the total number of states is the apparent topological spacetime dimension 4 multiplied by $\bar{\alpha}_0$. Finally, the number of elementary particles in the "complete" standard model is determined from the cohomology of $K(E_\infty)$ as $N(SM) = |E_\infty E_\infty|(\mathbf{16})/\mathbf{128} \simeq \bar{\alpha}_0/\mathbf{2}$ as should be.

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